Application of Theoretical Growth Functions for Mongolian Oak (*Qurcus Mongolica*)

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Abstract Four alternative functions are used for fitting tree height and diameter growth models for mongolian oak. (*Quercus mongolica* Fisch. et Turcz.). The data set includes 1250 random trees and 755 dominant trees coming from 510 temporary plots. The resultsshow that the Richards function is the best model for predicting height, diameter at breast height (DBH) and dominant height from age. The average growth curve of dominant height is used as a guide curve for the construction of a site index table which is partially validated using an independent data set. The Mitscherlich function is the best model for estimating height and dominant height from DBH.

Keywords: Growth functions, Richards function, Site index, Mongolian oak, *Quercus mongolica*)

Introduction

Mongolian oak(*Quercus mongolica* Fisch. et Turcz.) stands constitute the largest type of the secondary forest in Heilongjiang Province, China(Hu, 1979). The 1991 statistics of the Administrative Bureau of Forest Resources for Heilongjiang Province indicated an estimated area of 514977 hm^2 for mongolian oak stands and a volume of 32618000 m^3 for this species. The study of growth and yield of this species is important in order to elaborate management guidelines, but such studies are still lacking.

The aim of this paper is to develop average height and diameter growth models for mongolian oak. We first make a short review of some common theoretical growth functions and we recall their mathematical properties. We then present the data set and compare the results obtained when fitting the different growth functions. Finally we construct and test a site index table for mongolian oak in Heilongjiang Province.

Data Collection

The area under study is the Mudanjiang region in Heilongjiang Province. It is located between longitude 128°51′ and 134°5′ E and between latitude 43°40′ and 49°25′ N. Elevations range from 400 to 800 m. The average annual temperature varies between 2.5°C and 4°C. The frost-free period is between 110 and 140 days and the mean annual precipitation between 460 and 690 mm·yr^{-1}.

Data were collected from 510 temporary plots located in 10 forestry bureaus of the Mudanjiang Forest Administrative Bureau in Heilongjiang Province. The plots were sampled according to a systematic design. All the plots come from oak dominated secondary high forest stands that are approximately even-aged.

2077 trees were sampled in these plots. 827 of these trees were selected among dominant trees whereas the 1250 others were selected randomly (some but few of them were dominant). 755 of the 827 dominant trees were used to fit the models and 72 were used to test them. Summary statistics are presented in Table 1. The measurements that were carried out for each tree are total age(A), diameter at breast height(DBH)and total height (H).

Table 1. General statistics for stand attributes of the 510 sample plots

Stand variables	Min.	Max.	Mean	Standard deviation	Coefficient of variation
stand age(yr.)	15	135	47.0	20.96	0.443
mean DBH(cm)	6	85	13.6	7.031	0.516
mean height(m)	4	24	10.2	3.127	0.307
stand volume(m^3/hm^2)	5	335	72.9	48.56	0.667

Stem analysis were carried out on 10 dominant trees that were selected and felled in different plots corresponding to different age and site classes. These data

were intended to validate qualitatively the models derived from the other data .

Methods

In recent years, the Richards function has been extensively used to fit tree and stand growth models (Pienaar et al. 1973; Causton & Venus 1981; Ito et al. 1985; Houllier et Leban 1991). This function was derived from a generalization of the von Bertalanffy growth function(Richards 1959). The integrated form of Richards function is:

$$Y = a\,[1-b\exp(-kt)]^{c} \qquad (1)$$

where Y is a tree or stand attribute(e.g. height, biomass): t is time or age; a, b, k and c are parameters:

a is the upper asymptotic value of Y and k is related to time scale: $a>0$, $k>0$;

b is related to the origin of the growth curve on the time axis while c determines the shape of the growth curve and the relative location of its inflection point along the Y axis. Possible combinations of b and c are(b >0 and $c>=1$) or ($b<0$ and $c<0$).

The mathematical properties of the Richards growth function can be obtained by deriving equation(1) (see Fig. 1) as follows:
Let Z be the current growth rate, Zr the current relative growth rate and Zm be the total mean increment function:

$$Z = \frac{dy}{dt} = abkc\exp(-kt)(1-b\exp(-kt))^{c-1}$$

$$Z = kcY((\frac{a}{Y})^{1/c}-1) \qquad (2)$$

$$Zr = kc((\frac{a}{Y})^{1/c}-1) \qquad (3)$$

$$Zm = \frac{Y}{t} = \frac{a}{t}(1-b\exp(-kt))^{c} \qquad (4)$$

Let Zmax be the maximum growth rate function, T_{max}(resp. Y_{max})denote the time(resp. the Y value) at which $Z=Z_{max}$:

$$Z_{max} = ak(\frac{c-1}{c})^{c-1} \qquad (5)$$

$$T_{max} = \frac{1}{k}\log(bc) \qquad (6)$$

$$Y_{max} = a(\frac{c-1}{c})^{c} \qquad (7)$$

Different values of b and c give rise to different functions(Jiang et al.1990a; Millier 1982). Some functions that are derived from the general Richards equation(1) and that are frequently used in forestry are the following(Li 1988, 1991):

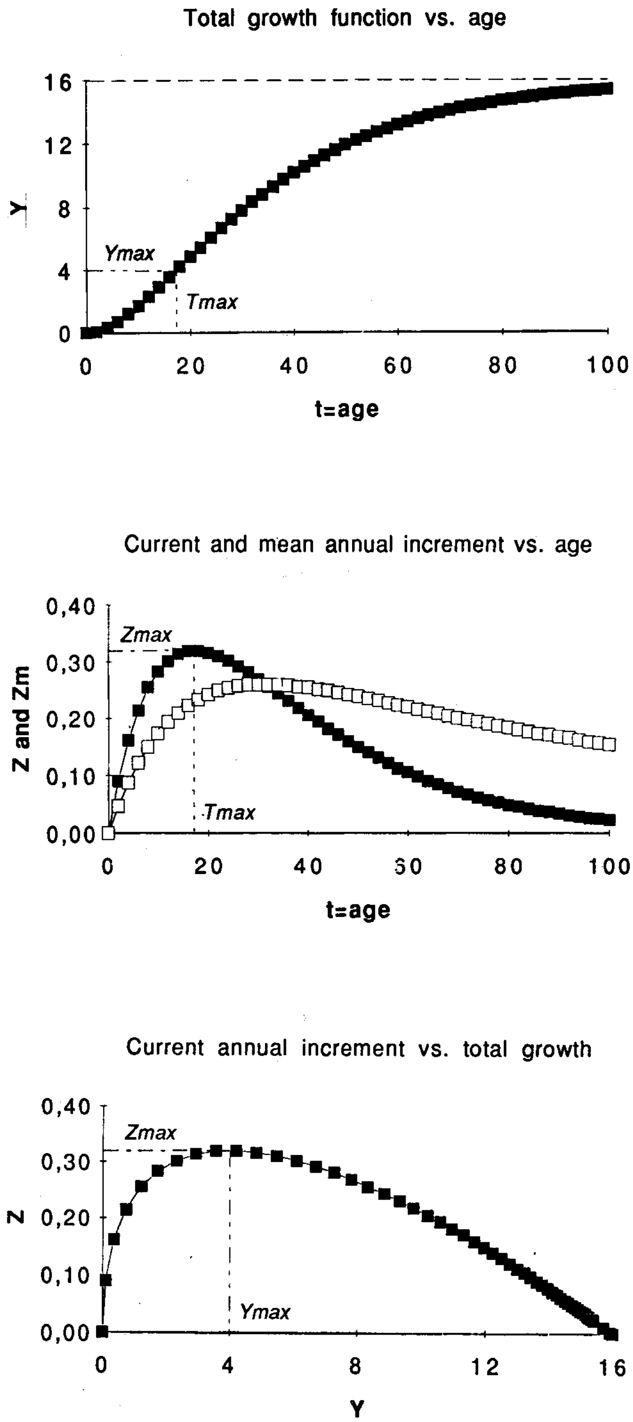

Fig. 1 Different aspects of the Richards growth function
Y = yield (= cumulated annual increments)
Z =current annual increment (= growth rate)
Zm = mean annual increment
a = 16; b=1; k=0.04; c=2

•the simplified Richards function obtained with c>1

and b=1 so that $Y_{(0)}=0$

$$Y = a(1-\exp(-kt))^c \tag{8}$$

•the Mitscherlich function obtained with c=1 and b>0 and which has no inflection point:

$$Y = a(1-b\exp(-kt)) \tag{9}$$

•the logistic function obtained with b = -b' < 0 and c=-1:

$$Y = \frac{a}{1+b'\exp(-kt)} \tag{10}$$

•the Gompertz function obtained as the limit when c tends to infinite:

$$Y = a\exp(-d\exp(kt)) \tag{11}$$

where d is a new parameter derived from a, b and k.

In this study we used these simplified growth functions [eq.(8) to (11)] to fit individual diameter and height growth data for mongolian oak. We also used these functions for predicting individual height from DBH.

The parameters were estimated using the NLIN procedure of SAS package(SAS 1988) and choosing the Marquardt method (Marquardt 1963). The models were fitted separately for the 1250 random trees and the 755 dominant trees. For equation (8) [resp.(9)], we systematically tested whether c (resp.b)was different from 1 we fixed it to 1 and we fitted the model again.

Results

Comparison of the different equations

The statistical results are presented in Table 2 (resp. Table 3) for the 1250 randomly selected trees (resp. 755 dominant trees). The four theoretical growth functions [eq.(8) to (11)] have approximately similar performances according to the following criteria: the pseudo-coefficient of determination(R^2)and the root mean squared error (RMSE). The simplified Richards function [eq. (8)] is the best model for predicting individual height, DBH and dominant height from age. The Mitscherlich function [eq.(9)] is the best model for predicting height and dominant height from DBH.

Construction of the site index table.

The proportional method(Jiang et al.,1990 b, Li et al., 1990) also called the Schumacher's or the guide curve method(Clutter et al.1983) was used to derive a set of height growth curve fitted to the simplified Richards function was used as the guide curve:

$$H_d = 16.04[1-\exp(-0.0408A)]^{1.976} \tag{12}$$

Table 2. Results of nonlinear adjustment of eq.(8) to (11) for the 1250 randomly selected trees.

Dependent variable	Independent variable	Equation	R^2	RMSE	Remark*
H	A	(8)	0.634	2.17m	c=1
H	A	(9)	0.634	2.17m	b=1
H	A	(10)	0.629	2.24M	
H	A	(11)	0.626	2.17	
Best function is: H = 15.08 [1 - exp (-0.0261A)]					
DBH	A	(8)	0.883	2.92 cm	
DBH	A	(9)	0.872	3.12 cm	b=1
DBH	A	(10)	0.881	2.93 cm	
DBH	A	(11)	0.860	3.18 cm	
Best function is: DBH = 145.0 [1 - exp(-0.0028)A]$^{1.0898}$					
H	DBH	(8)	0.674	2.07m	
H	DBH	(9)	0.694	2.00m	
H	DBH	(10)	0.697	2.01m	
H	DBH	(11)	0.670	2.09m	
Best function is: H = 19.64 [1- 0.9442 exp(-0.0489 DBH)]					

* In equations(8)[resp.(9)], when c(resp.b) was not significantly different from 1, its value was arbitrarily set to 1 and the model was fitted again.

Table 3. Results of nonlinear adjustment of eq.(8)to (11) for the 755 dominant trees.

Dependent variable	Independent variable	Equation	R^2	RMSE	Remark*
H	A	(8)	0.687	2.00m	
H	A	(9)	0.684	2.06m	
H	A	(10)	0.682	2.01m	
H	A	(11)	0.654	2.10m	
Best function is: H = 16.04 [1- exp(-0.0408A)]$^{1.976}$					
DBH	A	(8)	0.879	3.30 cm	
DBH	A	(9)	0.869	3.43 cm	b=1
DBH	A	(10)	0.878	3.32 cm	
DBH	A	(11)	0.857	3.58 cm	
Best function is: DBH = 152.0 [1- exp(-0.0028A)]$^{[illegible]}$					
H	DBH	(8)	0.621	2.28 m	
H	DBH	(9)	0.633	2.25 m	b=1
H	DBH	(10)	0.624	2.26 m	
H	DBH	(11)	0.619	2.29 m	
Best function is: H = 19.20[1 - 1.0084 exp(-0.0413 DBH)]					

* In equations (8) [resp. (9)], when c(resp.b) was not significantly different from 1, its value was arbitrarily set to 1 and the model was fitted again.

50 years was selected as the base age and the width of site index classes was set to 2 m. The site index

table of mongolian oak is presented in Table 4 and the associated growth curves are drawn on Figure 2 .

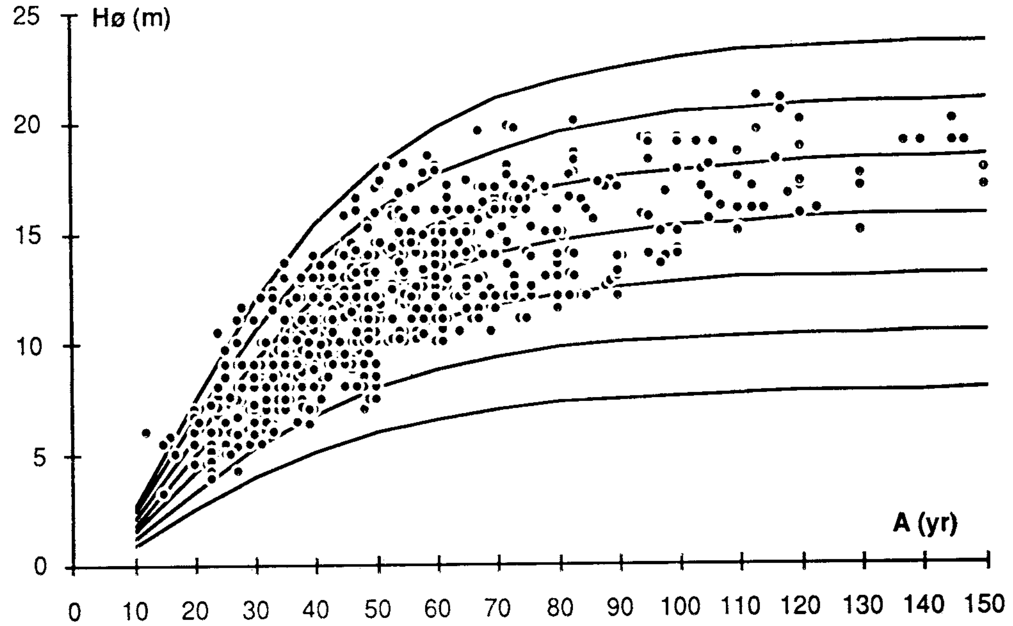

Fig. 2 Height growth curves of dominant mongolian oaks in Heilongjiang Province: data and theoretical curves

Table 4. Site index table for mongolian oak: dominant height growth for different site classes.

Aage	Site class: height at 50 years						
	6	8	10	12	14	16	18
10	0.91	1.21	1.52	1.82	2.12	2.43	2.73
20	2.49	3.32	4.16	4.99	5.82	6.65	7.48
30	3.97	5.29	6.62	7.94	9.27	10.59	11.91
40	5.14	6.85	8.57	10.28	11.99	13.71	15.42
50	6.00	8.00	10.00	12.00	14.00	16.00	18.00
60	6.61	8.81	11.01	13.22	15.42	17.62	19.82
70	7.03	9.37	11.71	14.06	16.40	18.74	21.09
80	7.32	9.75	12.19	14.63	17.07	19.51	21.95
90	7.51	10.01	12.52	15.02	17.52	20.02	22.53
100	7.64	10.19	12.73	15.28	17.82	20.37	22.92
110	7.73	10.30	12.88	15.45	18.03	20.60	23.18
120	7.78	10.38	12.97	15.57	18.16	20.76	23.35
130	7.82	10.43	13.04	15.65	18.26	20.86	23.47
140	7.85	10.47	13.08	15.70	18.32	20.93	23.55
150	7.87	10.49	13.11	15.73	18.36	20.98	23.60

Validation

We first validated qualitatively the model by comparing graphically the data coming from the stem analysis and the site index curves(Figure 3).

We then tested the accuracy of the average guide curve with 72 dominant trees which had not been used for constructing the site index table. We applied the following method(see Lang et al.,1989). We estimated dominant height H_ϕ* from age with eq. (12).We then fitted the following linear model:

$$H_\phi = a + b\, H_\phi^* \qquad (13)$$

and got: a=0.1435, b=0.9954, R=0.81, n=72, average relative error=-0.76% .

We then calculated the following F-statistic:

$$F = \frac{n-2}{2}\,\frac{na^2 + 2a(b-1)\sum_{i=1}^{n} H_{\phi i}^* - (b-1)^2\sum_{i=1}^{n} H_{\phi i}^{*2}}{H_{\phi i} - a - b\sum_{i=1}^{n} H_{\phi i}^*)^2} \qquad (14)$$

where: H_ϕ = height of dominant trees, H_ϕ*= estimated dominant height, n = number of sample trees. a and b are the parameters of equation(13). We got: F=0.6996< $F_{2,70}(0.05)$ = 3.1277. The average height growth curve is therefore validated and the site index table is suitable for forestry practice .

Conclusion

The above results show that the Richards equation is suitable for describing average height and diameter growth as functions of age .These functions were successfully fitted to randomly selected trees and to dominant trees . Applying the proportional method, the average dominant height growth function was then used as a guide curve in order to derive a site index table .

It must however be recalled that this method is very sensitive to the sampling design and, more precisely,

that its reliability depends on the balance between age and site classes. When the data are unbalanced, this method may yield biased results(often, the height growth of older stands is underestimated because there is a lack of good sites in the older part of the sample). The validation that we carried out in this paper does not address this problem and id therefore only partial. Solving this problem would require true dynamic data: stem analysis, permanent plots or short-term increments(Houllier, 1990). Unfortunately, only few such data were available in the context of this study: we just had stem analysis data coming from 10 trees. Figure 2 indicates that the theoretical height growth curves are qualitatively consistent with the observed growth curves (we only represented the four oldest trees for which stem analysis data were available). Clearly, this data set should now be completed in order to perform a fully satisfying quantitative validation .

The relationship between DBH and height is not a true growth relationship. Nevertheless we also applied and fitted the usual growth functions to study this allometric static relationship and we found that the Mitscherlich function is the best model for predicting tree height from DBH .

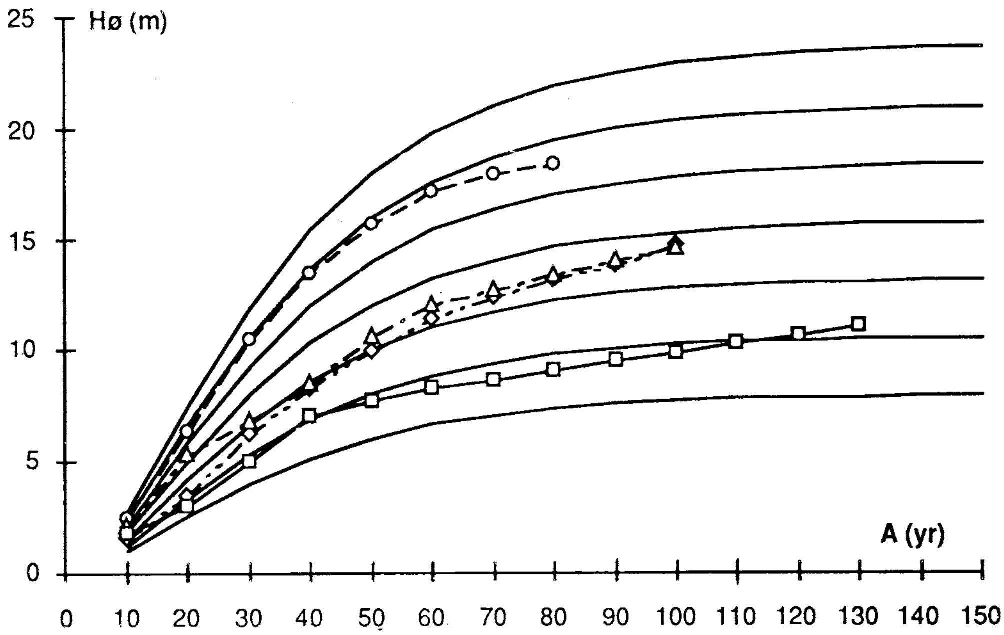

Fig. 3 Height growth of mongolian oak in Heilongjiang Province: theoretical curves compared to stem analysis data. The 4 oldest trees are represented.

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